Supplemental Material: Chiral degeneracies and Fermi-surface Chern numbers in bcc Fe

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I. CHIRAL TOUCHING BETWEEN FERMI SHEETS INDUCED BY MAGNETIZATION PRECESSION

Figure S1 shows the Fermi contours of bands nine and ten of bcc Fe on the ΓNH ($k_z = 0$) plane in Fig. 3, calculated with the magnetization pointing along [001]. The blow-up on the right shows the two points of contact between sheets 9 and 10₂, located along a degeneracy loop protected by mirror symmetry (Sec. VI.B.1). Those gluing points render ill-defined the Chern numbers of the low-symmetry pockets (10₂, 10₃, 10₄, 10₅), as discussed in Sec. VI.B.3.



FIG. S1. Left: Fermi contours of bands nine and ten on the Γ NH Brillouin-zone slice at $k_z = 0$, with the magnetization pointing along [001] (polar angle $\theta = 0$). Right: Detail showing the gluing points between sheets 9 and 10₂ along a nodal ring. The solid-line portion of the nodal ring is below the Fermi level, and the dashed-line portions are above.

In Fig. S2 the magnetization has been tilted by 20° towards the [100] axis, breaking the mirror symmetry. As a result the nodal rings have been reduced to a few Weyl points, and the previously glued-together Fermi sheets became isolated, with well-defined Chern numbers given by the enclosed chiral charges (Sec. VI.B.3). For example, pocket 10_2 has Chern number +1, because it encloses a single touching point of negative chirality with the band below.



FIG. S2. Left: Fermi contours of bands nine and ten on the Γ NH Brillouin-zone slice at $k_z = 0$, with the magnetization tilted by 20° towards the [100] axis. Right: Detail showing the now-detached sheets 9 and 10₂. The nodal ring has evaporated, leaving behind a few remnant Weyl points represented by the colored disks, with chiralities χ .



FIG. S3. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S4. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S5. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S6. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S7. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S8. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S9. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S10. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S11. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S12. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S13. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S14. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S15. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S16. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S17. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S18. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .



FIG. S19. Upper panel: Fermi contours of bands nine and ten, calculated with the magnetization tilted by 20° (the azimuthal angle ϕ is indicated by the dashed red line in the lower panel). The remnant Weyl points are displayed as colored disks, and the evaporated nodal ring is shown as a guide to the eye. Lower panel: Chern number of pocket 10_2 versus ϕ .

II. CHIRAL TOUCHING BETWEEN FERMI SHEETS UPON VARYING THE FERMI LEVEL

The results in this section were obtained with the magnetization along the easy axis [001]. Figure S20 shows the Fermi contours of bands nine and ten on the ΓNP ($k_x = k_y$) plane in Fig. 3. Pockets 9 and 10_1 have zero Chern number, and pockets 10_6 and 10_7 have Chern numbers -1 and +1 (see Table III). The series of snapshots in Figs. S21-S25 depicts the touching event between sheets 9 and 10_7 upon increasing the Fermi level, leading to a transfer of Chern number between them (Sec. VI.C.2).



FIG. S20. Left: Fermi contours of bands nine and ten on the Γ NP Brillouin-zone slice at $k_x = k_y$, evaluated for the true (unshifted) Fermi level. Right: Detail showing the region of closest approach between sheets 9 and 10₇.



FIG. S21. Upper-left panel: Energy bands along the line Δ in Fig. 3, close to the electron pocket 10_7 [see also Fig. 13(a)]. Energies are measured from the true Fermi level. Upper-right panel: Chern number of pocket 10_7 versus the Fermi-level shift. Lower panel: Fermi contours inside the red square in Fig. S20 for $\Delta E_F = 0.000$ eV (the dashed red line in the upper panels). The colored disks represent Weyl nodes between bands nine and ten, with chiral charges χ .



FIG. S22. Upper-left panel: Energy bands along the line Δ in Fig. 3, close to the electron pocket 10₇ [see also Fig. 13(a)]. Energies are measured from the true Fermi level. Upper-right panel: Chern number of pocket 10₇ versus the Fermi-level shift. Lower panel: Fermi contours inside the red square in Fig. S20 for $\Delta E_F = 0.035$ eV (the dashed red line in the upper panels). The colored disks represent Weyl nodes between bands nine and ten, with chiral charges χ .



FIG. S23. Upper-left panel: Energy bands along the line Δ in Fig. 3, close to the electron pocket 10_7 [see also Fig. 13(a)]. Energies are measured from the true Fermi level. Upper-right panel: Chern number of pocket 10_7 versus the Fermi-level shift. Lower panel: Fermi contours inside the red square in Fig. S20 for $\Delta E_F = 0.070$ eV (the dashed red line in the upper panels). The colored disks represent Weyl nodes between bands nine and ten, with chiral charges χ .



FIG. S24. Upper-left panel: Energy bands along the line Δ in Fig. 3, close to the electron pocket 10_7 [see also Fig. 13(a)]. Energies are measured from the true Fermi level. Upper-right panel: Chern number of pocket 10_7 versus the Fermi-level shift. Lower panel: Fermi contours inside the red square in Fig. S20 for $\Delta E_F = 0.105$ eV (the dashed red line in the upper panels). The colored disks represent Weyl nodes between bands nine and ten, with chiral charges χ .



FIG. S25. Upper-left panel: Energy bands along the line Δ in Fig. 3, close to the electron pocket 10_7 [see also Fig. 13(a)]. Energies are measured from the true Fermi level. Upper-right panel: Chern number of pocket 10_7 versus the Fermi-level shift. Lower panel: Fermi contours inside the red square in Fig. S20 for $\Delta E_F = 0.140$ eV (the dashed red line in the upper panels). The colored disks represent Weyl nodes between bands nine and ten, with chiral charges χ .