Origin of the asymmetric light emission from molecular exciton–polaritons

TOMÁŠ NEUMAN¹ AND JAVIER AIZPURUA¹,²,*

¹Centro de Física de Materiales de San Sebastián, CFM–MPC (CSIC-UPV/EHU), Paseo Manuel Lardizabal 5, 20018 Donostia-San Sebastián, Spain
²Donostia International Physics Center (DIPC), 20018 San Sebastián-Donostia, Spain
*Corresponding author: aizpurua@ehu.eus

Molecular emitters located in an optical cavity are known to experience a dramatic modification of the energy and dynamics of their light emission, establishing novel routes for the generation of non-classical states of light. Under monochromatic illumination, spectral asymmetries in cavity-enhanced molecular fluorescence often emerge due to the formation of hybrid polaritonic states (upper and lower polaritons). By applying the theory of open-quantum systems, we show that under strong-coupling conditions, it is essential to account for the interaction of the molecular electronic states with their vibrational environment (dephasing reservoir) to address the complex dynamics of light emission. The interaction with the dephasing reservoir yields a transfer of energy between the polariton states, favoring the transition toward the lower polariton. As a result, we show that the inelastic light emission originates mainly from the lower polariton state regardless of the pumping laser frequency, thus producing asymmetric light emission spectra. Furthermore, we show that, when several molecules are considered, intermolecular coupling can break the symmetry of the system, enabling originally dark polaritons to emit light, as revealed in the fluorescence spectrum by the emergence of new emission peaks. These results stress that accounting for the interaction with dephasing reservoirs is key to interpret molecular light emission in cavities, consistent with experimental observations.

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1. INTRODUCTION

The interaction of light with molecules has attained much attention due to its potential in the photochemical reactivity of molecules [1], the generation of non-classical states of light [2,3], molecular spectroscopy [4,5], chemical fingerprinting, or in the fundamental investigation of single-molecule properties [2,6–12]. This interaction can be enhanced when a molecule is placed in an optical cavity. To that end, a large variety of optical cavities have been developed, ranging from Fabry–Perot resonators of macroscopic sizes to nanoscale plasmonic cavities where squeezed light is concentrated at (sub)nanometric scales and can interact efficiently with fundamental excitations (e.g., molecular excitons or vibrations) of only a few molecules.

When placed in optical (plasmonic) cavities, excitons in organic molecules can strongly interact with the cavity modes and form new mixed exciton–photon (plasmon) excitations, so-called exciton–polaritons [6,13–42]. Exciton–polaritons have been broadly analyzed in connection with their fluorescence properties, cavity-induced (photo)chemistry [31,32,43–47], polariton lasing and polariton condensation [48–56], and polariton-mediated energy transfer [57,58].

Molecular emitters located in an optical cavity are known to experience a dramatic modification of the energy and dynamics of their light emission, establishing novel routes for the generation of non-classical states of light. Under monochromatic illumination, spectral asymmetries in cavity-enhanced molecular fluorescence often emerge due to the formation of hybrid polaritonic states (upper and lower polaritons). By applying the theory of open-quantum systems, we show that under strong-coupling conditions, it is essential to account for the interaction of the molecular electronic states with their vibrational environment (dephasing reservoir) to address the complex dynamics of light emission. The interaction with the dephasing reservoir yields a transfer of energy between the polariton states, favoring the transition toward the lower polariton. As a result, we show that the inelastic light emission originates mainly from the lower polariton state regardless of the pumping laser frequency, thus producing asymmetric light emission spectra. Furthermore, we show that, when several molecules are considered, intermolecular coupling can break the symmetry of the system, enabling originally dark polaritons to emit light, as revealed in the fluorescence spectrum by the emergence of new emission peaks. These results stress that accounting for the interaction with dephasing reservoirs is key to interpret molecular light emission in cavities, consistent with experimental observations.
quantum master equation [67] that describes the spectral asymmetries observed experimentally in the polariton emission and action (excitation) spectra [13,30,61]. We show that the dominant emission from the lower polariton state is a consequence of the interaction between the excitons and the dephasing reservoir, which in principle includes the effects of both the internal molecular vibrations and the solvent.

2. OPEN QUANTUM SYSTEM THEORY OF (COLLECTIVE) EXCITON-CAVITY-MODE COUPLING

We describe the molecules as two-level electronic systems composed of the ground state, \(|g\rangle\), and the excited state, \(|e\rangle\), interacting with their respective reservoirs, including both the internal molecular vibrational modes and the fluctuations of the local environment of each molecule [66]. The local environment of a molecule is responsible for the electronic dephasing processes [e.g., vibrations of the molecule or the environment [62,63,68-72], fluctuations of solvent polarization, etc., as presented schematically in Fig. 1(a)]. The two-level excitonic term of the Hamiltonian of the \(i\)th molecule is

\[ H_{c,e} = \hbar \omega_e \sigma_i^0 \sigma_i^e, \tag{1} \]

where \(\hbar \omega_e\) is the excitonic energy, and \(\sigma_i^0, \sigma_i^e\) are the two-level-system lowering and raising operators between the many-body excited state, \(|e_i\rangle\), and the many-body ground state, \(|g_i\rangle\), of the \(i\)th molecule. Each molecule interacts with its local dephasing reservoir described by the Hamiltonian

\[ H_{\text{res},i} = \hbar \omega_{e,i} \sigma_i^0 \sigma_i^e, \tag{2} \]

via the exciton-dephasing reservoir interaction Hamiltonian

\[ H_{\text{e-res},i} = d_{R,i} \sigma_i^e \sigma_i^0 (\sigma_i^e + \sigma_i^0). \tag{3} \]

Here \(\sigma_{i}^0, \sigma_{i}^e\) are the bosonic annihilation operators of the collective reservoir mode \(i\) interacting locally with the exciton of the \(i\)th molecule [65,70,72-76]. \(\dagger\) stands for the Hermitian conjugate. We have assumed that the reservoir modes have the same frequency of \(\Omega_{R,e,i} = \Omega_{R,g,i} = \Omega_R\) in the excited state (\(\Omega_{R,\sigma_i}\)) and the ground state (\(\Omega_{R,\sigma_i}\)). The equilibrium position of the reservoir mode is rigidly displaced in the electronic excited state of the \(i\)th molecule by a dimensionless constant \(d_R\) with respect to its equilibrium position in the ground electronic state. As we describe below, we further consider that the reservoir modes are phenomenologically broadened.

The intermolecular excitonic interactions are assumed to be described through the Hamiltonian

\[ H_{c-e} = \sum_{ij} G_{ij} \sigma_i^e \sigma_j^e + H.c., \tag{4} \]

where \(G_{ij}\) are coupling constants that generally depend on the spatial distribution of the individual molecules as well as on their mutual orientations.

The molecular excitons interact with a single bosonic cavity mode of frequency \(\omega_c\):

\[ H_c = \hbar \omega_c a^\dagger a + H.c., \tag{5} \]

where \(a(a^\dagger)\) is the bosonic annihilation (creation) operator of the cavity mode. The \(i\)th molecule interacts with the cavity mode via the coupling Hamiltonian

\[ H_{c-e,i} = \hbar g_i \sigma_i^e a + H.c., \tag{6} \]

where \(g_i\) is the respective cavity-mode-exciton coupling constant.

The total Hamiltonian describing the cavity and molecular excitations finally becomes

\[ H_{\text{tot}} = H_c + H_{c-e} + \sum_i (H_{\text{e-res},i} + H_{\text{e-res,c}} + H_{c-e,i}). \tag{7} \]

The Hamiltonian \(H_{\text{tot}}\) contains information on the coherent dynamics of the system, but also accounts for the coupling of molecular excitons with their respective dephasing reservoirs. Importantly, \(H_{\text{tot}}\) does not account for exciton decay and photon leakage. To properly account for this, we describe the dynamics of the system via the master equation for the density matrix, \(\rho\), including the effects of the environment via the phenomenological Lindblad terms of the form

\[ \mathcal{L}_\Theta(\rho) = \frac{\Gamma}{2} (2O_i^\dagger \rho O_i - \{O_i^\dagger O_i, \rho\}), \]

with \(O_i\) the operator of the respective excitation, the phenomenological damping constants of the respective excitations \(\gamma_{\Theta_i}\), and...

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**Fig. 1.** Effect of dephasing processes on the light emission from a cavity mode strongly coupled with a single exciton. (a) Schematic representation of a molecule interacting with its dephasing bath containing internal molecular vibrations but also environmental degrees of freedom such as fluctuating polarization of the solvent molecules. The bath modes are represented by bosonic annihilation operators \(B_i\). (b) Schematic-level diagrams of an exciton in a cavity that is decoupled (left) and after the coupling is turned on (right) within the MBM. The cavity-exciton coupling gives rise to new polariton states, \(|+\rangle\) and \(|-\rangle\), and opens new incoherent decay paths between \(|+\rangle\) and \(|-\rangle\) with respective states \(\gamma_{+e} > \gamma_{-e}\) . (c) Level diagram indicating the intermolecular population transfer between the polariton states as in panel (b) but for the JCM where the rates \(\gamma_{+e}\) and \(\gamma_{-e}\) are equal \((\gamma_{+e} = \gamma_{-e} = \sin^2 \Theta \cos^2 \phi\)). (d) The bath spectral density \(j(\omega)\) is given by Eq. (22) for the parameters \(\hbar \gamma_R = 400 \text{ meV}\), \(\hbar \Omega_R = 400 \text{ meV}\), and \(\gamma_R = 0.173\) [for which \(\hbar J(0) \approx 20 \text{ meV}\)]. Calculations of selected emission and absorption spectra for smaller values of \(\Omega_R\) are shown in Supplement 1. The vertical lines indicate the positions where the spectral density is evaluated to obtain the values of the Markovian decay rates \(\gamma_{+e}, \gamma_{-e}, \gamma_{+c}\), and \(\gamma_{-c}\).
with \{ ,\} the anti-commutator. The quantum master equation that includes all the necessary Hamiltonian and Lindblad terms becomes

\[ \dot{\rho} = \frac{1}{i\hbar} [H_{\text{tot}}, \rho] + \sum_i \mathcal{L}_i (\rho), \]

where \( \mathcal{L}_i \) depends on the model under consideration. As we detail in what follows, the dynamics encompassed in Eq. (8) leads to the asymmetries observed in the optical response of the strongly coupled system.

3. STRONG COUPLING OF A SINGLE-MOLECULE EXCITON WITH A CAVITY MODE

In the strong-coupling regime, the plasmon–exciton interaction \( g_i = g \) becomes so significant that it overcomes the intrinsic electronic (\( \gamma_g = \gamma_o \)) and cavity (\( \gamma_d \)) decay rates and leads to the formation of new hybrid states, polaritonic states. The simplest situation arises when a single cavity mode couples with a single two-level electronic system in the single-excitation manifold, whose only the bare states \( |g, 0\rangle, |e, 0\rangle, \) and \( |g, 1\rangle \) are considered, with 0 (1) the number of cavity excitations (we omit the index \( i \) when talking about a single molecule). The new polaritonic eigenstates \( |+\rangle \) and \( |−\rangle \) become the coherent admixtures of the exciton and the cavity excitation depending on the magnitude of the coupling strength and the detuning of their respective frequencies:

\[ |+\rangle = \cos \theta |e, 0\rangle + \sin \theta |g, 1\rangle, \]
\[ |−\rangle = -\sin \theta |e, 0\rangle + \cos \theta |g, 1\rangle, \]

\[ \tan(2\theta) = \frac{2g}{\omega_0 - \omega_c} \quad \text{and} \quad 0 < 2\theta < \pi. \]

The scheme of the newly arising energy level structure is drawn schematically in Fig. 1(b) [and 1(c)]. The operators of the three-level system consisting originally of the states \( |0\rangle = |g, 0\rangle, \) \( |2\rangle = |e, 0\rangle, \) and \( |3\rangle = |g, 1\rangle \) can be more conveniently expressed in the new basis \( \{|0\rangle, |+\rangle, |−\rangle\} \) with help of Eq. (9). Most importantly, the operator \( \sigma^i \sigma \) responsible for the interaction with the dephasing reservoir in \( H_{\text{osc,i}} = H_{\text{cav}} \) becomes (approximated in the single-excitation subspace)

\[ \sigma^i \sigma \approx |2\rangle \langle 2| = \cos^2 \theta |+\rangle \langle +| + \sin^2 \theta |−\rangle \langle −| - \sin \theta \cos \theta |−\rangle \langle +| + |+\rangle \langle −|. \]

We further introduce the simplifying notation \( \sigma_{\xi \zeta} = |\xi\rangle \langle \zeta|, \) with \( \xi, \zeta \in \{+, −\} \) and rewrite the electron-reservoir coupling Hamiltonian as

\[ H_{\text{e-res}} = \hbar d_R \Omega_R \sigma^+ \sigma^- (B^\dagger + B) = \hbar d_R \sigma F \]
\[ = \hbar \cos^2 \theta \sigma^+ + \sin^2 \theta \sigma^- - \sin \theta \cos \theta (\sigma^+ + \sigma^+), \]

where we have defined \( F = d_R \Omega_R (B^\dagger + B). \)

Following the standard procedure [67], we now eliminate the dephasing reservoir and derive the incoherent dynamics of the strongly coupled system. To that end, we notice that the operators \( \sigma^+ + \sigma^−, \sigma^− + \sigma^+, \) and \( \sigma^− \) are eigen-operators of the polaritonic Hamiltonian \( H_{\text{pol}} = H_e + H_c + H_{\text{osc}} \) (eigen-operator \( O \) of the Hamiltonian \( H_{\text{pol}} \) defined as \( [H_{\text{pol}}, \hat{O}] = i\lambda \hat{O}, \) with \( \lambda \) a complex number), and in the interaction picture of \( H_{\text{pol}}, \) these operators have the following time dependences:

\[ \sigma^+ = \sigma^+ (\omega_0 - \omega_c)t, \]
\[ \sigma^− = \sigma^− (\omega_0 - \omega_c)t, \]
\[ \sigma^+ = \sigma^+ (\omega_0 - \omega_c)t, \]
\[ \sigma^− = \sigma^− (\omega_0 - \omega_c)t, \]

with \( \mathcal{O}(0) \) the Schrödinger-picture operators, and

\[ \alpha_{\pm} = \frac{\alpha_0 + \omega}{2} \pm \sqrt{\frac{(\omega_0 - \omega)^2}{4} + \frac{\alpha_0^2}{4}} \]

the frequencies of the upper \( (\omega_+) \) and lower \( (\omega_−) \) polaritons.

In the secular approximation, the incoherent processes are represented by the Lindblad terms describing the dephasing of the polariton states, \( \mathcal{L}_{\sigma^+} (\rho) \), the decay of \( |+\rangle \) to \( |−\rangle \), \( \mathcal{L}_{\sigma^-} (\rho) \), and the reverse process, \( \mathcal{L}_{\sigma^-} (\rho) \). For brevity, we have defined \( \sigma_{\sigma^+ + \sigma^−} = \cos^2 \theta \sigma^+ + \sin^2 \theta \sigma^- \). The respective dephasing and decay rates, \( \gamma_{\phi} = \gamma_{\sigma^+ + \sigma^-}, \gamma_{\sigma^+}, \) and \( \gamma_{\sigma^-} \), are determined from the properties of the dephasing reservoir characterized by its spectral density \( f(\omega) \):

\[ \gamma_{\sigma^+} = \cos^2 \theta \sin^2 \theta f(\omega_+ - \omega_c), \]
\[ \gamma_{\sigma^-} = \cos^2 \theta \sin^2 \theta f(\omega_- - \omega_c), \]
\[ \gamma_{\phi} = f(0). \]

The spectral density of the reservoir [65,70,72,74–76] is obtained as the Fourier transform of the reservoir’s two-time correlation function \( \langle F(t+s)F(t) \rangle \) [67]:

\[ f(\omega) = 2\Re\{ \int_0^\infty d\tau e^{i\omega \tau} \langle F(t+s)F(t) \rangle \}, \]

where \( \Re \{ \} \) is the real part. In particular, \( f(\omega) \) emerging from Eqs. (2) and (3) together with the Lindblad term \( \mathcal{L}_{\sigma^0} (\rho) \) (damped harmonic-oscillator reservoir [65,75]) calculated for zero temperature, \( T = 0 \) K, is

\[ f(\omega) = \frac{2\gamma_{\sigma^0} \Omega^2_R}{(\Omega_R - \omega)^2 + \gamma^2_R}. \]

The spectral density, \( f(\omega) \), of the considered vibrational bath [Eq. (22)] is shown in Fig. 1(d). \( f(\omega) \) has the form of a broad Lorentzian peak positioned at the positive side of the frequency axis. This stems from the condition \( T = 0 \) K, for which the polariton decay can result only in the spontaneous generation of excitations (vibrations) in an otherwise unpopulated reservoir. We note that for \( T > 0 \) K (a situation not considered in this paper), when the reservoir acquires thermal population, processes including absorption of a thermal reservoir excitation (appearing for negative \( \omega \)) would also contribute to \( f(\omega) \) [72,77–80]. The model parameters used in our study are specified in the caption of Fig. 1. As \( f(\omega) \) is not symmetrical with respect to the zero frequency, the transition \( |+\rangle \rightarrow |−\rangle \) given by the rate \( \gamma_{\sigma_-} = \cos^2 \theta \sin^2 \theta f(2|g|) \) is favored compared with the \( |−\rangle \rightarrow |+\rangle \) transition occurring with a rate of \( \gamma_{\sigma_+} = \cos^2 \theta \sin^2 \theta f(2|g|) \) [indicated by vertical lines in Fig. 1(d)]. We stress that this asymmetry is a general property of dephasing reservoirs and robustly appears in a wide range of non-Markovian dephasing models [70,73,74,76]. This imbalance of transfer of energy between the polariton states gives rise to the asymmetries observed in the emission spectra [13,30,61] that we address below.
Photon emission spectra normalized to the incident laser intensity $|E|^2$ as a function of excitation frequency $\omega_s$ within (a) the explicit-bath model, (b) the Markovian-bath model, and (c) the Jaynes–Cummings model. In all the calculations, we have considered the parameters $\hbar\omega_0 = \hbar\omega_c = 2$ eV, $\hbar\gamma_c = 150$ meV, $\hbar\gamma_a = 2 \times 10^{-2}$ meV, and $\hbar g = 100$ meV. The pure dephasing constant for JCM is $\gamma_\phi = J(0)$. The parameters of the bath are $\hbar\gamma_R = 400$ meV, $\hbar \omega_R = 400$ meV, and $\alpha_R = 0.173$.

Last, in strong coupling, we employ the polariton Lindblad operators $\mathcal{L}_\sigma(p)$ and $\mathcal{L}_\sigma(p)$ ($\sigma_+ = |0\rangle\langle+|$ and $\sigma_- = |0\rangle\langle-|$), where the decay rates of the upper, $\gamma_{\sigma_+}$, and the lower, $\gamma_{\sigma_-}$, polaritons are defined, respectively, as

$$\gamma_{\sigma_+} = \gamma_a \sin^2 \theta,$$

$$\gamma_{\sigma_-} = \gamma_a \cos^2 \theta,$$

where $\gamma_a$ is the decay rate of the bare cavity decoupled from the molecules. The phenomenological Lindblad terms $\mathcal{L}_\sigma(p)$ and $\mathcal{L}_\sigma(p)$ can be related to the commonly assumed phenomenological Lindblad super-operator describing the decay of the bare cavity, $\mathcal{L}_c(p)$. Under the strong-coupling condition, we write the photon annihilation operator $a$ in terms of the polariton operators $\sigma_+$ and $\sigma_-$ (in the single-excitation subspace):

$$a \approx \sin \theta \sigma_+ + \cos \theta \sigma_-, \quad (25)$$

and apply the secular approximation. Under such conditions, the Lindblad super-operator $\mathcal{L}_c(p)$ approximately transforms into a pair of the Lindblad terms, $\mathcal{L}_\sigma(p)$ and $\mathcal{L}_\sigma(p)$:

$$\mathcal{L}_c(p) \approx \mathcal{L}_\sigma(p) + \mathcal{L}_\sigma(p). \quad (26)$$

We also phenomenologically include the intrinsic molecular losses via $\mathcal{L}_\sigma(p)$, considering $\gamma_\sigma \ll \gamma_a$.

4. POLARITON EMISSION SPECTRA UNDER COHERENT DRIVING CONDITIONS

A. Single Molecule in a Cavities

In what follows, we consider several different approaches to the implementation of dephasing due to the reservoir. First, we explicitly implement the dephasing reservoir defined by $H_{\text{res},j} = H_{\text{res}}$ and $H_{\text{res},d} = H_{\text{res}}$ [Eqs. (2) and (3), respectively] and $\mathcal{L}_a(p)$ into the master equation as part of the simulated system [the explicit-bath model (EBM)] and solve for the spectral response (see Supplement 1 for details of the implementation of the reservoir degrees of freedom). In the second approach, we approximate the EBM and eliminate the dephasing reservoir from Eq. (8) using the Born–Markov and secular approximations, as described in the previous section, and introduce the effective dephasing and damping terms via the Lindblad super-operators $\mathcal{L}_\sigma(p)$, $\mathcal{L}_\sigma(p)$, $\mathcal{L}_\sigma(p)$, $\mathcal{L}_\sigma(p)$, $\mathcal{L}_\sigma(p)$, $\mathcal{L}_\sigma(p)$ [the Markovian-bath model (MBM)]. The effective rates are depicted schematically in Fig. 1(b).

As a third approach, we consider the commonly adopted Jaynes–Cummings model (JCM) where the effective dephasing and decay rates are first defined for the exciton of the molecule and the bare cavity mode, which are mutually decoupled. Note that this is in contrast with the MBM where the incoherent dynamics is derived in the polariton basis. The decays of the cavity and the molecular exciton are described in the JCM by $\mathcal{L}_\sigma(p)$ and $\mathcal{L}_\sigma(p)$, respectively, as defined earlier, and the pure dephasing is implemented via

$$\mathcal{L}_\sigma(p) = \frac{\gamma_a}{2} (2\sigma^\dagger \sigma \sigma^\dagger \sigma - \{\sigma^\dagger \sigma, \rho\}). \quad (27)$$

In the JCM, the interaction with the reservoir given in Eqs. (2) and (3) is not considered. Upon transformation into the polariton basis, the dephasing term in the JCM yields (among others) the interaction terms between $|+\rangle$ and $|-\rangle$, with equal rates for the $|+\rangle \rightarrow |-\rangle$ and $|-\rangle \rightarrow |+\rangle$ transitions, as depicted schematically in Fig. 1(c).

As we are interested in the response of the system under illumination by a monochromatic laser light, we introduce the driving term

$$H_{\text{pump}} = E(a^\dagger e^{i\omega t} + a e^{i\omega t}), \quad (28)$$

where $E$ is the amplitude of the laser pumping and $\omega$ is the laser frequency. We make sure that the pumping amplitude is small enough to conform with the single-excitation approximation.

We calculate the absorption spectra, $s_\lambda(\omega)$, of the system (assuming only the cavity interacts with the radiation field) and the inelastic emission spectra, $s_e(\omega; \omega_0)$, for different frequencies $\omega$ of the incident pumping laser. The spectra are calculated from the quantum regression theorem as one-sided Fourier transforms of the two-time correlation functions (more details are provided in Supplement 1):

$$s_\lambda(\omega) = 2\Re \int_0^\infty \langle a(\tau) a(0) \rangle \omega e^{i\omega \tau} d\tau, \quad (29)$$

$$s_e(\omega_0; \omega) = 2\Re \int_0^\infty \langle a^\dagger(\tau) a(0) \rangle \omega e^{i\omega \tau} d\tau, \quad (30)$$

where the double-angle brackets are defined as $\langle a^\dagger(\tau) a(0) \rangle = \lim_{\tau \rightarrow -\infty} \langle a^\dagger(\tau) a(0) \rangle$, $\lambda = \{a^\dagger(\tau) a(0) \rangle \lambda = \langle a^\dagger(\tau) a(0) \rangle \lambda = \lim_{\tau \rightarrow -\infty} \langle a^\dagger(\tau) a(0) \rangle \lambda$.  

\[\text{Research Article} \]
The calculated emission spectra for the reservoir spectral density assumed in Fig. 1(d) are shown in Fig. 2 within both the EBM and MBM, and are compared with the result obtained from the JCM. To simplify the discussion, in what follows we concentrate on the special case when the energies of the plasmonic and excitonic transitions are matched ($\omega_{\text{c}} = \omega_{0}$). In Figs. 2(a)–2(c), we plot the emission spectra of the strongly coupled single-molecule exciton with the cavity mode as a function of excitation frequency $\omega_{0}$. Within (a) the EBM, (b) the MBM, and (c) the JCM. For both the EBM and MBM, the color maps offer the same qualitative and very similar quantitative results. The inelastic emission arises mainly from the transition of the lower polariton to the ground state and thus leads to a clear dominance of the emission peak of the lower polariton. Contrarily, the JCM yields a fully symmetrical result independently of the excitation frequency, which contradicts the experimental evidence [13,30,61]. The implementation of the dephasing in the JCM is thus unable to correctly describe the imbalance in the dephasing-driven population transfer between the polaritonic states.

**B. Many Molecules in a Cavity**

The strong coupling between a single-molecule exciton and a cavity mode is fundamentally important; however, in realistic systems, the cavity is usually coupled to several molecular samples [81]. We therefore extend our description to cavities containing $N$ molecules and calculate the absorption and emission spectra as defined in Eqs. (29) and (30), respectively, using the EBM. In the EBM, we include the Lindblad terms $\mathcal{L}_{\text{B}}(\rho)$, $\mathcal{L}_{\text{A}}(\rho)$, $\mathcal{L}_{\text{S}}(\rho)$, and $\mathcal{L}_{\text{E}}(\rho)$, where $S_{+} = |0\rangle\langle +|$ and $S_{-} = |0\rangle\langle -|$ (with $|0\rangle$ the ground state and $|+\rangle$ $|-\rangle$) the upper (lower) polariton branches. We consider $\gamma_{\text{A}} = \gamma_{\text{B}} = \gamma_{\text{E}}$. More details are provided in Supplement 1. Like in the single-molecule case, the strong coupling of the cavity mode with the excitons of many molecules gives rise to the upper (|+) and lower (|-) polariton branches, as depicted schematically in Fig. 3(a). In addition to the bright polaritons $|+\rangle$ and $|-\rangle$, there are $N-1$ states that are decoupled from the cavity [if the intermolecular interaction in Eq. (4) preserves the equivalence of all the molecules] and are commonly called dark polaritons $|D_{i}\rangle$. The polariton states incoherently couple via the dephasing reservoir, which allows population transfer among the bright polaritons $|+\rangle$ and $|-\rangle$ and the dark $|D_{i}\rangle$ polaritons $\gamma_{S_{+}S_{+}} = \gamma_{S_{-}S_{-}} = \gamma_{S_{+}S_{-}}$, and $\gamma_{S_{+}S_{+}}$, with decay rates of $\gamma_{\text{A}} = \gamma_{0} = \gamma_{\text{E}} = \gamma_{\sigma}$ for the dark polaritons, and $\gamma_{S_{+}}$ and $\gamma_{S_{-}}$ for the bright polaritons. The incoherent processes can be included into the system dynamics via the Lindblad terms $\mathcal{L}_{\text{O}}(\rho)$, with the respective rates of $\gamma_{\text{O}}$, and $\mathcal{O}_{i} \in \{S_{+}S_{+}, S_{+}S_{-}, S_{-}S_{+}, S_{+}, S_{-}, S_{+}, S_{-}\}$. In our notation, $S_{i} = |0\rangle\langle i|$ (with $|0\rangle$ the ground state). The newly arising states also undergo dephasing (not shown in the schematics), in analogy with the case of the single exciton. More details on the respective processes are provided in Supplement 1. As opposed to the case where only a
single molecule is considered, in the collective scenario, the dark polariton states mediate the population decay \(|+\rangle \rightarrow |−\rangle\) and change the population dynamics observed for the lower polariton state if \(|+\rangle\) is pumped.

In the following, we assume an intermolecular coupling of the form
\[
G_{ij} = \frac{G_0}{|i - j|^3} \quad \text{for} \ i \neq j \quad \text{and}
\]
\[
G_{ij} = 0 \quad \text{for} \ i = j,
\]
and set
\[
\hbar G_0 = \frac{p_0^2}{4\pi\varepsilon_0 r_0^4},
\]
with \(p_0 = 0.2 \cdot \text{e} \cdot \text{nm}\) the transition dipole moment of the exciton; \(r_0 = 2 \text{ nm}\) the effective intermolecular distance; and \(\varepsilon_0\) the vacuum permittivity. This choice of \(G_{ij}\) describes a set of interacting molecules whose dipoles are arranged along a line (e.g., in x direction) with a constant spacing of \(r_0\) and with parallel dipole moments \(p_0\) (e.g., oriented along z). The intermolecular interaction given by Eq. (31) weakly perturbs the polariton structure given by the collective-mode-exciton Hamiltonian; however, it breaks the symmetry of the Hamiltonian (makes the molecules inequivalent). Due to this symmetry breaking, the originally dark polariton states \(|D_i\rangle\) couple with the cavity mode and become observable in the spectra. We discuss more details of the collective-coupling model in Supplement 1. We note that the symmetry of the system Hamiltonian can be broken in different ways, for example, by introducing disorder into the system.

As an illustrative example, we calculate the emission and absorption spectra of four mutually interacting molecules that are coupled to the cavity with \(\hbar g_j = \hbar g_0 = 100 \text{ meV}\). The system is pumped at an upper polariton frequency of \(\hbar \omega_1 = 2.2 \text{ eV}\). The result is shown in Fig. 3(b) for \(N = 4\) molecules interacting with the cavity mode. The emission spectrum (black solid line) shows a dominant peak originating from the lower polariton \(|−\rangle\) (appearing at \(\approx 1.8 \text{ eV}\)) as in the single-molecular case. Another sharp emission peak of low intensity, which was not present in the single-molecule case, emerges at a frequency around that of the decoupled molecules, i.e., \(\approx 2 \text{ eV}\). This new peak is a signature of the polariton states \(|D_i\rangle\) that are dark in the collective-coupling model where the excitons do not interact directly among themselves, but become bright after introducing the intermolecular coupling in Eq. (4). Experiments where large numbers of molecules couple with the cavity show that the photoluminescence peak of the dark polariton can have comparable intensity to the emission peaks of the lower polaritons [30,61]. On the other hand, the absorption spectrum (blue dashed line) features two absorption peaks of commensurate intensity at the frequencies of the \(|+\rangle\) and \(|−\rangle\) polariton branches. As a result of the inter-polariton transfer induced by the reservoir, the lower-polariton peak has slightly higher spectral intensity and is narrower than the upper-polariton peak, since the latter is broadened by the decay processes induced by the dephasing reservoir [72].

Finally, in Figs. 3(c)–3(f) we present two-dimensional maps containing the emission (vertical axis) and excitation (horizontal axis) spectra of systems containing \(N = 2\) [Fig. 3(c)], \(N = 3\) [Fig. 3(d)], \(N = 4\) [Fig. 3(e)], and \(N = 5\) [Fig. 3(f)] molecules (considering \(\hbar g = 100 \text{ meV}\)). The emission pattern is in all the cases similar to that in the single-molecule case [Figs. 2(a) and 2(b)], exhibiting a doublet of emission peaks originating from \(|+\rangle\) and \(|−\rangle\) that are split by the collectively enhanced coupling \(\sqrt{N} \omega_0\). Between the \(|+\rangle\) and \(|−\rangle\) polariton peaks, in this collective scenario, there appears an additional feature corresponding to the dark polaritons in both the emission and excitation spectra, which is hardly distinguishable in the spectral maps. The dominance of the lower-polariton peak in all calculated spectra is in accordance with the mechanism of incoherent population transfer in strongly coupled systems discussed above. We can observe that the inelastic emission from the lower polariton branch is the most efficient when the upper polariton is pumped. In this case, the interaction with the reservoir efficiently incoherently populates \(|−\rangle\), which in turn emits the inelastic photons. We now briefly analyze the polariton dynamics in the collective scenario that gives rise to the asymmetry of the inelastic photon emission.

5. POLARITON DYNAMICS IN THE COLLECTIVE SCENARIO

We have shown that the dephasing reservoir gives rise to incoherent transitions between the polariton states that preferentially lead from the states of higher energy toward the states of lower energy \((|+\rangle \rightarrow |D_j\rangle, |+\rangle \rightarrow |−\rangle, \text{and} |D_j\rangle \rightarrow |−\rangle\). This phenomenology has been addressed in detail by other authors [72,77–80]. Here we briefly complete the discussion of our model and focus on the dynamics of these decay processes, and calculate the time evolution of the polariton populations \(n_j = (S_j^\dagger S_j)\), \(n_i = (S_i^\dagger S_i)\), and \(n_D = \frac{1}{N(N-1)} \sum_{j \neq i} (S_j^\dagger S_j)\) assuming that the populations evolve according to the master equation [Eq. (8) with Eq. (26)] that explicitly includes the dephasing reservoir (the EBM). We compare the EBM population dynamics with the dynamics of a rate-equation model (REM) based on the diagram of levels and decays displayed in Fig. 3(a) (more details are provided in Supplement 1). In the REM, only the incoherent dynamics of the populations of the respective states is studied (the population decay) and processes related with pure dephasing are not considered. We calculate the dynamics assuming the upper polariton is initially fully populated, \(n_+ = 1\) and \(n_− = n_D = 0\), and then decays spontaneously (the coherent driving [Eq. (28)] is switched off) to the ground state \(\{0\}\) and to the other polariton states, namely, \(|−\rangle\) and \(|D_j\rangle\).

In Fig. 4, we plot the polariton populations on a logarithmic scale as a function of time obtained from the numerical time evolution of the full system-reservoir density matrix (EBM—full lines) together with the solution of the REM (full lines) for \(N = 4\) molecules [Fig. 4(a)] and \(N = 1000\) molecules [Fig. 4(b)] (using the REM only). For \(N = 4\), the REM matches well with the EBM, with only slight deviations from the exact population dynamics. \(n_+\) (black) exhibits a rapid decay at a total rate of \(\gamma_{n_+} = \gamma_{S_+} + \gamma_{S_S} + (N−1)\gamma_{S_D^S}\) from its original population to the ground state, \(\{0\}\) (\(\gamma_{S_+}\)), but also to the lower polariton, \(|−\rangle\) (\(\gamma_{S_S^S}\)), and the dark polaritons, \(|D_j\rangle\), at a rate of \((N−1)\gamma_{S_D^S}\) that pumps the lower (\(n_−\), red) and dark polariton (\(n_D\), blue) populations. After this initial impulse, the dark polariton population starts to steadily decay to the ground state (\(\gamma_{S_S}\)) and to the lower-polariton state (\(\gamma_{S_S^S}\)). Similarly to the dark polaritons, the lower polariton first gets populated due to the fast-decaying upper polariton. After that, \(|−\rangle\) rapidly decays to the ground state (\(\gamma_{S_+}\)), but only until it reaches the regime when
\begin{align*}
n_\text{e} \text{ is dominantly pumped by the slowly decaying dark polariton (}\gamma_{S_jS_0}\text{). In this regime, the decay of } n_\text{e} \text{ becomes limited by the pumping and resembles that of the dark polaritons [the bottleneck effect; red lines in Fig. 4(a)].}

\text{In Supplement 1, we show that the decay rates connecting the polariton states are inversely proportional to the number of molecules, } \gamma_{S_jS_0} \propto 1/N. \text{ Since the upper and lower polaritons in our model decay fast to the ground state (} \gamma_{S_j} \propto \gamma_0\text{) regardless of } N, \text{ the initial stages of their respective population dynamics are practically independent of the number of molecules. However, as } N \text{ is increased, the rate of decay of the dark polariton to the lower polariton becomes progressively smaller (} \gamma_{S_jS_0} \propto 1/N \text{) until it becomes fully limited by the intrinsic rate } \gamma_\text{e} \text{ for } N \to \infty. \text{ This tendency is apparent from Fig. 4(b) where we plot the population decay for } N = 1000 \text{ molecules as obtained from the REM.}

\text{Finally, we remark that the model described in this paper is capable of addressing the dynamics of population transfer among polaritonic states, but does not explain the long lifetime of the lower polariton state that has been reported in the literature [62,82–84]. In our approach, the terminal slow decay of } n_\text{e} \text{ arises due to a bottleneck in the form of a slowly decaying dark polariton state. The explanation of the long lower-polariton lifetime requires further modeling of the microscopic decay mechanisms of the coupled cavity mode and the molecular excitons.}

\textbf{6. CONCLUSION}

\text{In conclusion, we have demonstrated that the dephasing reservoir in strongly coupled cavity-mode-exciton systems can lead to asymmetries in the observed emission spectra, favoring the light emission from the lower polariton and suppressing the emission from the upper polariton. The asymmetry in the inelastic light emission from the cavity arises naturally from the model, which explicitly considers the dephasing arises as an effective damped-harmonic oscillator. The coupling with the reservoir in the strong-coupling regime naturally favors the transfer of the population of higher-energy polaritons toward the polaritons of lower energy (} |\text{+}\rangle \to |D_0\rangle, |\text{+}\rangle \to |\rangle\text{, and } |D_0\rangle \to |\rangle\text{), including the dark polaritons if many molecules are considered. This process leads to the prevalence of the inelastic photon emission from the lower polariton } |\rangle \text{ and a considerably shorter lifetime of the upper polariton } |\text{+}\rangle. \text{ Moreover, if many mutually interacting molecules are coupled to the cavity, the dark polariton states can become bright and give rise to a new peak in the polariton emission spectrum. This new peak is then positioned approximately at the frequency of the uncoupled excitons, which is consistent with the experimental observations [13,30,61].}

\text{The results presented in this paper provide an intuitive view of the processes that stand behind the experimental observations and can serve as guidelines for future implementations of dephasing in strongly coupled systems.}

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\text{See Supplement 1 for supporting content.}

\textbf{REFERENCES}


