An experimental method for studying two-dimensional percolation

Gustavo Ariel Schwartz[a)]
Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón 1, Buenos Aires (1428), Argentina and Applied Physics Department, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

Silvio J. Ludueña
Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón 1, Buenos Aires (1428), Argentina

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A simple experimental technique for analyzing a broad range of two-dimensional percolation problems is presented. The method is based on a combination of the use of a CAD program capable of dealing with a variety of site-bond combinations and an electrical measurement of conductance. The latter is achieved by printing the computer generated pattern using conducting ink. The metal-insulator transition is measured on the print out of the lattice, and the conductivity critical exponent and the percolation threshold are calculated from these measurements.
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I. INTRODUCTION

Percolation is involved in a broad range of physical phenomena including forest fires, propagation of epidemics, oil fields, wafer-scale integration, ferromagnetism, and diffusion in disordered media.[1–6] One of the easiest and most interesting percolation-related phenomena to study is the electrical behavior of a system of conducting particles dispersed in an insulating matrix. These disordered conductor-insulator composites show a variety of incompletely understood phenomena, some of which have important technological applications.[7–9] The electrical behavior of these systems can be described by percolation models in which the conducting sites are distributed in a spatially random fashion in an insulating medium. When the volume fraction of the conducting material, p, is lower than a critical volume fraction p_c, no macroscopic conducting pathway exists, and the composite remains in the insulating phase. When the volume fraction is greater than the critical volume fraction, the system becomes electrically conducting. Near the transition, the electrical conductivity σ of these composites is described by the relation[1]

$$\sigma = \begin{cases} 0 & \text{if } p < p_c \\ \sigma_0 (p - p_c)^t & \text{if } p \geqslant p_c, \end{cases} \tag{1}$$

where p is the volume fraction of the conductive phase, p_c is the percolation threshold, and t is the critical exponent. The power law behavior in Eq. (1) is expected to hold in a narrow range of concentrations above the percolation threshold. For two-dimensional systems the exponent t is universal meaning that it depends only on the dimensionality of the system and not on its small-scale details. This universality means that t is the same for different lattice geometries and even for continuum systems. However, for three-dimensional systems, experimental values of the exponent t for continuum systems are often higher than those obtained from numerical simulation of lattice networks.[10]

Although the idea of percolative phase transitions is conceptually simple, few analytical solutions for the critical parameters have been found. Moreover, it is difficult to determine these parameters for real systems such as conducting particles in an insulating matrix. The problems arise in part from the difficulty of keeping the different variables under control (due to, for example, a poorly known volume fraction and particle size distribution and possible nonrandomness).

In previous works, the value of the conductivity exponent t was established by experimental measurements. Last and Thouless[11] and later Mehr *et al.*[12] determined the bulk conductivity of a sheet of colloidal graphite paper with holes randomly punched in it. Watson and Leath[13] performed a similar experiment cutting links in a metal wire mesh. The values $t = 1.1 \pm 0.3$ and $t = 1.38 \pm 0.12$ were obtained in Refs. 12 and 13, respectively. Later, Dubson and Garland[10] used a computer-controlled digital $x-y$ plotter to cut a percolating pattern from sheets of aluminized plastic. As a pattern was cut, a digital ohmmeter continuously monitored the resistance of the sheet. From their experiment they estimated $t = 1.29 \pm 0.03$.

A wide variety of numerical simulations has established the value of the conductivity exponent t with increasing precision.[14–19] The most efficient and accurate of the numerical simulations are based on the algorithm of Lobb and Frank.[20,21] The presently accepted value of the conductivity exponent, derived from numerical simulations, is $t = 1.30 \pm 0.01$ in two dimensions.

We present an experimental technique to study percolation by a simple method that can be carried out by undergraduate physics students. In our case, the percolation pattern to be analyzed is generated with a computer program that allows flexibility in particle shape, size, and location. In this way it is possible to compare computer calculations and experimental results. Because the patterns are computer generated, the samples are well characterized with none of the ambiguities sometimes inherent in experimental studies. We will analyze only one case to show the reliability of the method, and leave open further work.

II. EXPERIMENTAL TECHNIQUES

We developed a computer program to draw small black particles over a white background. The particles are placed, with probability p, on a 64×64 lattice. The program has a

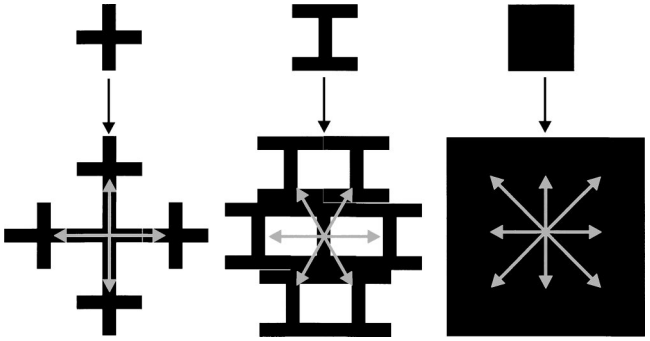

Fig. 1. Some particle shapes used by the program. (a) Only nearest neighbors on a square lattice. (b) Only nearest neighbors on a triangular lattice. (c) First and second neighbors on a square lattice.

graphical interface in which the user selects the lattice geometry, the particle shape, and the volume fraction of the conductive filler. Figure 1 shows different particle shapes and their near-neighbor connections on different lattice geometries. When the layout is ready, it is printed on a Hewlett Packard 692C ink jet printer. Then two lines, the upper and lower ''electrodes,'' are painted with silver paint. Because the ink is conducting, we can measure the electrical resistance directly from the printed layout with a Hewlett Packard 34401A multimeter. To avoid the problems arising from variations of the ink level or the paper type, the program draws two continuous lines near the layout (see Fig. 2). The resistance of the two lines is measured and averaged. The resistance of the layout is then normalized by dividing by this mean. Although the resistance value of these control lines (as well as that of the layouts itself) depends on the ink level, paper type, and room conditions, the normalized resistance value for each layout is almost constant, with an error smaller than 3%, independent of the above-mentioned conditions. The contact resistance between the silver paint and the ink is negligible compared to the resistance of the layout, and it was not taken into account in the analysis.

III. RESULTS AND DISCUSSION

To introduce the method and determine its reliability, we will analyze the case of a square lattice with nearest neighbors. The experiment was performed by plotting cross shape particles [see Fig. 1(a)] on a square lattice. Figure 2 shows a typical layout at $p=0.35$. Note how some clusters (sets of particles that are in electrical contact) start to appear, but no percolating cluster exists at this stage. Also we note that each particle has no more than four neighbors. Twenty-four layouts were printed and measured from $p=0.58$ to $p=0.95$. To get an estimate for p_c and the critical exponent t, we fitted the normalized conductivity data for $p>p_c$ to Eq. (1) by varying p_c in the interval from 0.53 to 0.60 in steps of 0.005. For each value of p_c, the value of t was determined from the slope of the normalized conductivity versus $(p-p_c)$ on a log–log plot. The lowest value of the root mean square error (for the critical exponent) was found for $p_c=0.585\pm0.005$ with a value of $t=1.28\pm0.07$ for the critical exponent.

These values are in close agreement with those calculated by numerical simulation as well as with previous experimental work (see Sec. I), showing the reliability of the method. Stauffer[22] and Jan[23] have calculated the most accurate value of p_c for site percolation with nearest-neighbors' connections

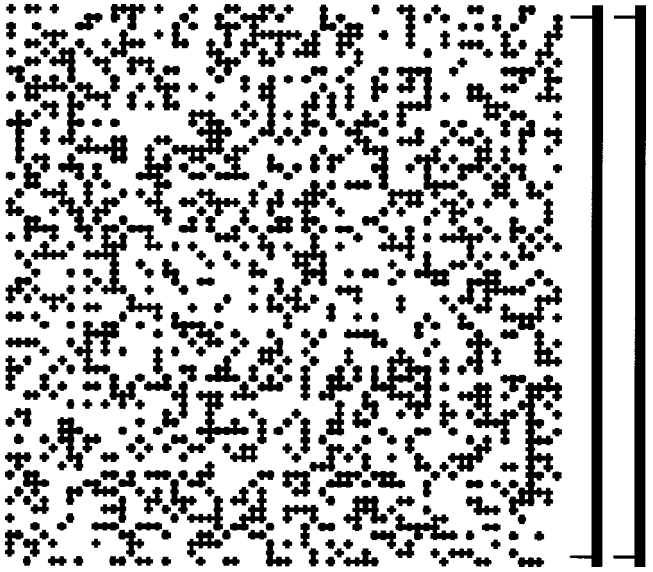

Fig. 2. A picture of the layout with conducting particles for concentration $p=0.35$. Some clusters begin to appear. The two lines on the right side are used to normalize the value of the resistance.

and obtained $p_c=0.592\,746\pm0.000\,001$. Lobb and Frank[20,21] found by numerical simulation a value of $t=1.30\pm0.01$. Figure 3 shows the experimental values with the best fit to Eq. (1). It is important to observe that the whole range of concentrations has been fitted by a power law. Although such a fit may apply for some particular systems due to their finite size, it does not hold in general.

One of the advantages of this method over previous ones is that the percolation pattern to be analyzed is generated and printed by a computer program. The program lets us detect the percolating cluster, analyze its backbone structure, and add or remove selected particles to explore different effects. The high resolution of the printer lets us work with arbitrary particle shapes and modify the number of neighbors, that is, the connectivity or coordination number. Also, we can change the lattice (or particle) size to analyze finite size effects.

There are several papers related to the problem described here. Schmelzer *et al.*[24] have analyzed finite-size effects in the conductivity of atomic clusters deposited between lithographically defined contacts with nanometer scale separations. Martin and Heaney[25] studied the unusually rapid increase in the resistivity of composites of carbon black particles in polyethylene, making this material commercially useful as current-limiting thermistors; and Han *et al.*[26] generated a random pattern of the Swiss cheese percolating system on a thin aluminum film, and analyzed the percolation threshold as well as the critical exponent.

IV. SUMMARY

We have described an experimental method for the demonstration of percolation. The method is simple and elegant and lets us control the shape and position of the conducting particles, the lattice geometry, and the connectivity between particles. The method is simple enough to be carried out in an undergraduate student laboratory.

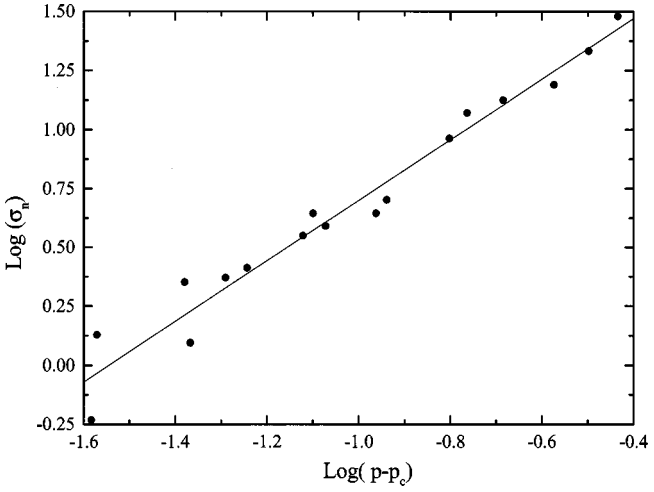

Fig. 3. Normalized conductivity values of cross shape particles on a square lattice as a function of $p-p_c$. The line represents the best fit to Eq. (1).

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[a)]Electronic mail: schwartz@fy.chalmers.se

[1]D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, Washington, DC, 1992), and references therein.

[2]C. M. Fortuin and P. W. Kasteleyn, ''Random-cluster model. 1. Introduction and relation to other models,'' Physica (Amsterdam) **57**, 536–564 (1972).

[3]H. L. Frisch and J. M. Hammersley, ''Percolation processes and related topics,'' J. Soc. Ind. Appl. Math. **11**, 894–918 (1963).

[4]J. T. Cox and R. Durrett, ''Limit-theorems for the spread of epidemics and forest fires,'' Stochastic Proc. Appl. **30**, 171–191 (1988).

[5]J. van den Berg, G. R. Grimmett, and R. B. Schinazi, ''Dependent random graphs and spatial epidemics,'' Ann. Appl. Probab. **8**, 317–336 (1998).

[6]J. W. Greene and A. El Gamal, ''Configuration of VLSI arrays in the presence of defects,'' J. Assoc. Comput. Mach. **31**, 694–717 (1984).

[7]V. E. Gul, *Structure and Properties of Conducting Polymer Composites* (V.S.P. International Science, Utrecht, The Netherlands, 1996).

[8]E. K. Sichel, *Carbon Black-Polymer Composites* (Dekker, New York, 1982).

[9]N. Probst, in *Carbon Black Science and Technology*, edited by J. B. Donnet, R. C. Bansal, and M. J. Wang (Dekker, New York, 1993), Chap. 8, pp. 271–288.

[10]M. A. Dubson and J. C. Garland, ''Measurement of the conductivity exponent in two-dimensional percolating networks: Square lattice versus random-void continuum,'' Phys. Rev. B **32** (11), 7621–7623 (1985).

[11]B. J. Last and D. J. Thouless, ''Percolation theory and electrical conductivity,'' Phys. Rev. Lett. **27**, 1719–1721 (1971).

[12]R. Mehr, T. Grossman, N. Kristianpoller, and Y. Gefen, ''Simple percolation experiment in 2 dimensions,'' Am. J. Phys. **54** (3), 271–273 (1986).

[13]B. P. Watson and P. L. Leath, ''Conductivity in the two-dimensional-site percolation problem,'' Phys. Rev. B **9** (11), 4893–4896 (1974).

[14]J. P. Straley, ''The ant in the labyrinth—diffusion in random networks near the percolation-threshold,'' J. Phys. C **13**, 2991–3002 (1980).

[15]R. Fisch and A. B. Harris, ''Critical behavior of random resistor networks near the percolation threshold,'' Phys. Rev. B **18** (1), 416–420 (1978).

[16]J. Adler, Y. Meir, A. Aharony, A. B. Harris, and L. Klein, ''Low concentration series in general dimension,'' J. Stat. Phys. **58**, 511–538 (1990).

[17]J. Adler, ''Conductivity exponents from the analysis of series expansions for random resistor networks,'' J. Phys. A **18**, 307–314 (1985).

[18]B. Derrida and J. Vannimenus, ''A transfer-matrix approach to random resistor networks,'' J. Phys. A **15**, L557–L564 (1982).

[19]J. G. Zabolitzky, ''Monte Carlo evidence against the Alexander-Orbach conjecture for percolation conductivity,'' Phys. Rev. B **30** (7), 4077–4079 (1984).

[20]C. J. Lobb and D. J. Frank, ''Percolative conduction and the Alexander-Orbach conjecture in two dimensions,'' Phys. Rev. B **30**, 4090–4092 (1984).

[21]C. J. Lobb and D. J. Frank, ''Highly efficient algorithm for percolative transport studies in two dimensions,'' Phys. Rev. B **37**, 302–307 (1988).

[22]D. Stauffer, ''Minireview: New results for old percolation,'' Physica A **242** (1–2), 1–7 (1997).

[23]N. Jan, ''Large lattice random site percolation,'' Physica A **266**, 72–75 (1999).

[24]J. Schmelzer, S. A. Brown, A. Wurl, M. Hyslop, and R. J. Blaikie, ''Finite-size effects in the conductivity of cluster assembled nanostructures,'' Phys. Rev. Lett. **88** (22), 226802-1–226802-4 (2002).

[25]J. E. Martin and M. B. Heaney, ''Reversible thermal fusing model of carbon black current-limiting thermistors,'' Phys. Rev. B **62** (14), 9390–9397 (2000).

[26]K. H. Han, Z. S. Lim, and S. I. Lee, ''Experimental study of the conductivity exponent in a two-dimensional Swiss cheese percolating network,'' Physica B **167** (3), 185–188 (1990).